[23]J. O. Hirschfelder, C. F. Curtiss, and R. B. Bird, *Molecular Theory of Gases and Liquids* (Wiley, New York, 1954); S. Kielich, Acta Phys. Polon. 22, 65 (1962).

[24]S. Chandra, G. K. Pandey, and V. K. Agrawal, J. Phys. Chem. Solids 30, 1644 (1969).

[25]A. F. Devonshire, Proc. Roy. Soc. (London) A153, 601 (1936).

[26]P. Sauer, Z. Physik 194, 360 (1966).

[27]The dipolar and octupolar transition probability for the low-barrier case is being calculated as a function of the barrier parameter. The results in this approximation may become the subject of another communication. It may be mentioned that the OH^- alkali halide systems do not fall in the low-barrier limit. For details see Ref. 10.

[28]H. B. Shore, Phys. Rev. 151, 570 (1966); see Ref. 7 and other papers referred to there.

[29]A. A. Maradudin, Rept. Progr. Phys. 28, 331 (1965).

[30]P. D. Mannheim, Phys. Rev. 165, 1011 (1968).

[31]K. Knop, G. Pfishev, and W. Kanzig, Phys. Kondensierten Materie 7, 107 (1968).

[32]L. D. Schearer and T. L. Estle, Solid State Commun. 4, 639 (1969).

[33]U. Bosshard, R. W. Dreyfus, and W. Kanzig, Phys. Kondensierten Materie 4, 254 (1964).

[34]R. D. Kirby and A. J. Sievers, Solid State Commun. 6, 613 (1968); R. J. Elliot, W. Hayes, G. D. Jones, H. F. MacDonald, and C. T. Sennet, Proc. Roy. Soc. (London) 289, 1 (1965).

[35]R. L. Pompi and V. Narayanamurti, Solid State Commun. 6, 645 (1968).

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Tunneling through a Barrier Containing a Pair of Interaction Impurities*

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A theoretical study of the tunneling phenomenon through a metal-insulator-metal junction containing paramagnetic impurities is made. The impurity spins are assumed to be interacting weakly with each other through a general indirect-exchange interaction. Expressions for conductance are obtained by perturbation theory up to third order treating the impurity-electron–conduction-electron interaction as a perturbation. The voltage and temperature dependence of the resulting expression of conductance is analyzed and compared with the recent experimental results on the type of junction considered here. Agreement between the experimental and theoretical results is satisfactory.

I. INTRODUCTION

The use of tunneling phenomena is one of the most powerful methods of investigating electronic states in metals, semimetals, and semiconductors. Electron tunneling is also useful in investigating the interaction between electrons and internal excitations of magnetic impurities in the insulating barriers. A complete account of tunneling in solids has been given by Duke.[1] While investigating tunneling through metal-insulator-metal junctions, Wyatt[2] found that these junctions exhibit anomalous behavior in the conductance as a function of the applied bias. In particular, he found that the conductance had a logarithmic voltage dependence and that the zero-bias conductance increased logarithmically with the decrease in temperature.

There have been various theoretical attempts to explain the zero-bias anomalies, notable are those of Kim,[3] Anderson,[4] Appelbaum,[5] Solyom and Zawadowski.[6] Perhaps the most successful theory is that of Anderson and Appelbaum who have considered the interaction of a single magnetic impurity with the conduction electrons. Like Kondo,[7] they used the second Born approximation in considering the s-d exchange interaction. Recently, Beal Monod[8] investigated the effect of a pair of interacting magnetic impurities on the conductivity of a simple metal. He showed that coefficient of $\ln|k_BT/2\epsilon_F|$ remains negative, as it was for a single impurity, but its absolute value decreases.

In what follows, the tunneling phenomenon is investigated, taking into account the weak interaction between a pair of magnetic impurities. We shall perform the calculation of the scattering amplitude by perturbation theory up to third order. The interaction between the impurity electrons and conduction electrons is described by the s-d exchange interaction, and for simplicity, each impurity is supposed to have total spin $\frac{1}{2}$. We assume the two spins $\vec{S}_1$ and $\vec{S}_2$ to be coupled by a general interaction W. The interaction W may be due to the Ruderman-Kittel-Kasuya-Yosida (RKKY)[9] interaction between the impurities via the conduction electrons, direct interaction, or indirect exchange interaction.

The tunneling Hamiltonian method, first used by Cohen, Phillips, and Falicov,[10] is followed in the present work. In Sec. II, we shall formulate the Hamiltonian of the problem. In Secs. III and

IV, the expressions for conductance in second- and third-order perturbation theory, respectively, are obtained. In Sec. V, the results so obtained are discussed in the light of experimental results.

II. FORMULATION OF HAMILTONIAN

We shall consider a metal-A-metal-oxide-metal-B junction. The insulating barrier (metal oxide) includes paramagnetic impurities. A small bias voltage V is applied between the two metals. This situation is shown in Fig. 1. Metal A is usually a transition metal, and metal B is a nontransition metal. The metals are in their normal state. The impurities are supposed to be confined to the left-hand side only.

The Hamiltonian of the system can be written as

$$H=\sum_i \frac{p_i^2}{2m}+\sum_i V(\vec{x}_i)+\tfrac{1}{2}\sum_{i\neq j} U(\vec{x}_i-\vec{x}_j)\ . \tag{1}$$

In second-quantized form it becomes

$$H=H'+H''\ , \tag{2a}$$

$$H'=\int \psi^*(\vec{x})\,[p^2/2m+V(\vec{x})]\,\psi(\vec{x})\,d\vec{x}\ , \tag{2b}$$

$$H''=\tfrac{1}{2}\int \psi^*(\vec{x})\,\psi^*(\vec{x}')\,U(\vec{x}-\vec{x}')\,\psi(\vec{x}')\,\psi(\vec{x})\,d\vec{x}\,d\vec{x}'\ , \tag{2c}$$

where

$$\psi(\vec{x})=\textstyle\sum_i a_i\,\psi_i^a(\vec{x})+\sum_i b_i\,\psi_i^b(\vec{x})\ , \tag{3a}$$

$$\psi^*(\vec{x})=\textstyle\sum_i a_i^*\,\psi_i^{a*}(\vec{x})+\sum_i b_i^*\,\psi_i^{b*}(\vec{x})\ . \tag{3b}$$

The $\{\psi_i^a(\vec{x})\}$ and $\{\psi_i^b(\vec{x})\}$ are a complete set of one-electron states in the regions a and b of Fig. 1, respectively. The states of interest, $\{\psi_i^a(\vec{x})\}$ and $\{\psi_i^b(\vec{x})\}$ are the Bloch states $\{\phi_{k\sigma}^a(\vec{x})\}$ and $\{\phi_{k\sigma}^b(\vec{x})\}$ on sides a and b, respectively, along with the states $\{\phi_{d\sigma}^1(\vec{x})\}$ and $\{\phi_{d\sigma}^2(\vec{x}-\vec{R})\}$ of the localized electrons. The impurities are supposed to be located at origin and at $\vec{R}$. We obtain for $\psi(\vec{x})$,

$$\psi(\vec{x})=\sum_{k,\sigma} a_{k\sigma}\,\phi_{k\sigma}^a(\vec{x})+\sum_{k'\sigma'} b_{k'\sigma'}\,\phi_{k'\sigma'}^b(\vec{x})+\sum_\sigma d_\sigma^1\,\phi_{d\sigma}^1(\vec{x})+\sum_\sigma d_\sigma^2\,\phi_{d\sigma}^2(\vec{x}-\vec{R})\ , \tag{4}$$

where $a_{k\sigma}$ and $b_{k\sigma}$ are destruction operators for an electron with wave vector $\vec{k}$ and spin $\vec{\sigma}$ on sides a and b, respectively, and d_σ is the destruction operator for an electron in a localized state. Substituting $\psi(\vec{x})$ and $\psi^*(\vec{x})$ in (2), where we have taken $U(\vec{x}-\vec{x}')$ as the appropriate electron-electron interaction, one obtains a Hamiltonian of the following forms:

$$H=H_1+H_2+H_3+\cdots\ , \tag{5}$$

$$H_1=\sum_{k,\sigma}\epsilon_{k\sigma}^a\,a_{k\sigma}^*\,a_{k\sigma}+\sum_{k,\sigma}\epsilon_{k\sigma}^b\,b_{k\sigma}^*\,b_{k\sigma}\ . \tag{6}$$

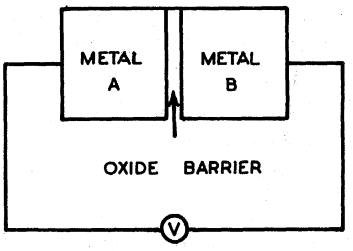

FIG. 1. Schematic representation of a tunnel junction.

This is just the single-particle conduction-electron Hamiltonian,

$$H_2=\sum_{k,k',\sigma}(T_{kk'}\,a_{k\sigma}^*\,b_{k'\sigma}+T_{k'k}\,b_{k'\sigma}^*\,a_{k\sigma}) \tag{7a}$$

$$+\sum_{k,\sigma}(1+e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})\,T_{kd}^a\,(a_{k\sigma}^*\,d_\sigma+d_\sigma^*\,a_{k\sigma}) \tag{7b}$$

$$+\sum_{k,\sigma}(1+e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})\,T_{kd}^b\,(b_{k\sigma}^*\,d_\sigma+d_\sigma^*\,b_{k\sigma})\ . \tag{7c}$$

The first term is due to direct overlap of the conduction-electron wave functions on sides a and b as they tail into the barrier. The second and third terms are due to the overlap of the localized d states wave functions with those of the conduction electrons of the a and b sides, respectively:

$$H_3=\textstyle\sum_\sigma E_d n_\sigma-W\vec{S}_1\cdot\vec{S}_2\ . \tag{8}$$

E_d is the appropriate single-particle energies for the localized electrons, which are taken to be zero.

Terms involving the product of four operators for conduction electrons only are contained in H_4. These terms have been already considered by Kim.[3]

Terms involving the product of four operators for conduction electrons on side a and localized electrons are contained in H_5. The terms of most interest to us are the exchange-scattering terms:

$$2\Bigg(\sum_{kk'\sigma\sigma'} W_{dk,dk'}\,d_\sigma^{1*}\,a_{k\sigma'}^*\,d_{\sigma'}^1\,a_{k'\sigma}+\sum_{kk'\sigma\sigma'} e^{i(\vec{k}-\vec{k}')\cdot\vec{R}}\,W_{dk,dk'}\,d_\sigma^{2*}\,a_{k\sigma'}^*\,d_{\sigma'}^2\,a_{k'\sigma}\Bigg)\ .$$

This can be also written as

$$\sum_{kk'\sigma\sigma'} W_{dk,dk'}(1+e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})\,(d_\sigma^1+d_\sigma^2)^*\,a_{k\sigma'}^*\,(d_{\sigma'}^1+d_{\sigma'}^2)\,a_{k'\sigma}$$

$$+\sum_{kk'\sigma\sigma'} W_{dk,dk'}(1-e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})\,(d_\sigma^1-d_{\sigma'}^2)^*\,a_{k\sigma'}^*\,(d_{\sigma'}^1-d_{\sigma'}^2)\,a_{k'\sigma}\ . \tag{9}$$

In all the remaining terms we factorize the two-body operators, and retain only terms of the form

$$\textstyle\sum V_{kd}^a(1+e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})\,(d_\sigma^*\,a_{k\sigma}+a_{k\sigma}^*\,d_\sigma)\ . \tag{10}$$

In H_6, there are terms involving the product of

four operators for conduction electrons on side b and the localized state. Since the coupling between electrons on side b and the localized electron is very small, we retain only a term which is first order in this coupling, the product of three electron operators for side b and one localized-electron operator. Again, one obtains

$$H_6=\sum_{k,\sigma} V'_{kd}(d^*_\sigma b_{k\sigma}+b^*_{k\sigma}d_\sigma)\ . \tag{11}$$

In H_7, there are terms in which conduction-electron operators for sides a and b along with the localized-electron operator appear. Among these, we retain only

$$H_7=\sum_{kk',\sigma\sigma'} W_{kd,k'd}(1+e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})\, a^*_{k\sigma}(d^1_{\sigma'}+d^2_{\sigma'})^*\, b_{k'\sigma'}(d^1_\sigma+d^2_\sigma)+\text{H. c.} \tag{12a}$$

$$+\sum_{kk',\sigma\sigma'} W_{kd,k'd}(1-e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})\, a^*_{k\sigma}(d^1_{\sigma'}-d^2_{\sigma'})^*\, b_{k'\sigma'}(d^1_\sigma-d^2_\sigma)+\text{H. c.} \tag{12b}$$

$$+\sum_{kk',\sigma\sigma'} W_{kd,k'd}(1+e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})\, a^*_{k\sigma}d^*_{\sigma'}d_{\sigma'}b_{k'\sigma}+\text{H. c.} \tag{13}$$

As we are concerned with paramagnetic impurities, we replace the d operators by spin operators in (9) and (12), obtaining

$$\begin{aligned}
&J_a\Big(\sum_{kk'}(1+e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})\,[(\vec{S}_1+\vec{S}_2)_z(a^*_{k\uparrow}a_{k'\uparrow}-a^*_{k\downarrow}a_{k'\downarrow})+(\vec{S}_1+\vec{S}_2)^+(a^*_{k\downarrow}a_{k'\uparrow})+(\vec{S}_1+\vec{S}_2)^-(a^*_{k\uparrow}a_{k'\downarrow})]\\
&+\sum_{kk'}(1-e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})\,[(S_1-S_2)_z(a^*_{k\uparrow}a_{k'\uparrow}-a^*_{k\downarrow}a_{k'\downarrow})+(S_1-S_2)^+(a^*_{k\downarrow}a_{k'\uparrow})+(S_1-S_2)^-(a^*_{k\uparrow}a_{k'\downarrow})]\Big)\\
&+T_{J_a}\Big(\sum_{kk'}(1+e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})\{(\vec{S}_1+\vec{S}_2)_z[(a^*_{k\uparrow}b_{k'\uparrow}+b^*_{k'\uparrow}a_{k\uparrow})-(a^*_{k\downarrow}a_{k'\downarrow}+b^*_{k'\downarrow}a_{k\downarrow})]+(\vec{S}_1+\vec{S}_2)^+(a^*_{k\downarrow}b_{k'\uparrow}+b^*_{k'\downarrow}a_{k\uparrow})\\
&+(\vec{S}_1+\vec{S}_2)^-(a^*_{k\uparrow}b_{k'\downarrow}+b^*_{k'\uparrow}a_{k\downarrow})\}+\sum_{kk'}(1-e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})\{(\vec{S}_1-\vec{S}_2)_z[(a^*_{k\uparrow}b_{k'\uparrow}+b^*_{k'\uparrow}a_{k\uparrow})\\
&-(a^*_{k\downarrow}b_{k'\downarrow}+b^*_{k'\downarrow}a_{k\downarrow})]+(\vec{S}_1-\vec{S}_2)^+(a^*_{k\downarrow}b_{k'\uparrow}+b^*_{k'\downarrow}a_{k\uparrow})+(\vec{S}_1-\vec{S}_2)^-(a^*_{k\uparrow}b_{k'\downarrow}+b^*_{k'\uparrow}a_{k\downarrow})\}\Big)\ .
\end{aligned} \tag{14}$$

The $\vec{s}\ (\vec{S}_1+\vec{S}_2)$ part will leave the total spin of the pair unchanged, whereas the part $\vec{s}\ (\vec{S}_1-\vec{S}_2)$ will allow transition from $|\vec{I}|=0$ to $|\vec{I}|=1$ and vice versa. As there is no external field present, the conductions due to spin-up and spin-down electrons are equal. For the same reason, the thermal average of the z component $\vec{M}$ of $\vec{I}$ and $(\vec{S}_1+\vec{S}_2)$ is zero, but one will need the Boltzmann probabilities for the pair to be in a singlet ($|I|=0$) or a triplet ($|I|=1$) state given, respectively, by

$$\begin{aligned}
P_{|I|=0}&=P_0=\frac{1}{1+3e^{W/k_BT}}\ ,\\
P_{|I|=1}&=P_1=\frac{e^{W/k_BT}}{1+3e^{W/k_BT}}\ .
\end{aligned} \tag{15}$$

In addition to Eq. (14), we have also the following terms:

$$\begin{aligned}
&T\sum_{kk',\sigma}(a^*_{k\sigma}b_{k'\sigma}+b^*_{k'\sigma}a_{k\sigma})\\
&+T_a(1+e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})\sum_{kk',\sigma}(a^*_{k\sigma}b_{k'\sigma}+b^*_{k'\sigma}a_{k\sigma})\ .
\end{aligned} \tag{16}$$

The first term of (16) is just (7a). The second term represents all the nonexchange mechanisms for the tunneling in which conduction electrons interact with impurities. Typical nonexchange mechanisms of this sort are given in (13). Such terms also result from the interference between (7b), (10), (7c), and (11).

As noted by Anderson,[5] the localized states act as a bridge between the exponentially tailing wave functions of the conduction electrons of the opposite sides of the junction, thus effectively decreasing the size of the barrier for those electrons which tunnel across the junction by means of the localized states. As the coupling constant varies exponentially with junction thickness, a decrease in the effective thickness of the junction by a few angstrom can make the tunneling, assisted by impurities, sufficiently greater than tunneling due to direct overlap of the conduction-electrons' states on sides a and b even for low concentration, and as such we have neglected the contribution due to later phenomena. So, finally, we can write down the total Hamiltonian as

$$H=H_0+H_T+H_R\ , \tag{17}$$

where

$$H_0=\sum_{k,\sigma}\epsilon^a_{k\sigma}a^*_{k\sigma}a_{k\sigma}+\sum_{k,\sigma}\epsilon^b_{k\sigma}b^*_{k\sigma}b_{k\sigma}-W(\vec{S}_1\cdot\vec{S}_2)\ , \tag{18}$$

$$\tilde{\epsilon}^a_{k\sigma} = \epsilon^a_{k\sigma} + eV \,. \tag{19}$$

$-W(\vec{S}_1 \cdot \vec{S}_2)$ is the barrier energy and its eigenvalues are

$$W_I = \tfrac{1}{2} W[\tfrac{3}{2} - I(I+1)], \tag{20}$$

with

$$\vec{I} = \vec{S}_1 + \vec{S}_2 \,, \qquad |\vec{I}| = 0 \text{ or } 1 \,. \tag{21}$$

H_T represents the transfer of electrons from left to right, while H_R gives reflection of conduction electrons from the impurities. We further divide H_T and H_R into two parts:

$$\begin{aligned} H_T &= H_{T1} + H_{T2} \,, \\ H_R &= H_{R1} + H_{R2} \,. \end{aligned} \tag{22}$$

H_{T1} and H_{R1} includes only elastic processes, i.e., $|\vec{I}|$ is unchanged, while H_{T2} and H_{R2} includes inelastic processes in which $|\vec{I}|$ changes by ± 1.

In view of Eqs. (14) and (16) and subsequent discussion, the expressions for H_{T1}, H_{R1}, H_{T2}, and H_{R2} can be written down as

$$\begin{aligned} H_{T1} = T_{J_a} \sum_{kk'} (1 + e^{i(\vec{k}-\vec{k}')\cdot\vec{R}}) \{ (\vec{S}_1 + \vec{S}_2)_z [(a^*_{k\uparrow} b_{k'\uparrow} - a^*_{k\downarrow} b_{k'\downarrow}) + (b^*_{k'\uparrow} a_{k\uparrow} - b^*_{k'\downarrow} a_{k\downarrow})] + (\vec{S}_1 + \vec{S}_2)^+ (a^*_{k\downarrow} b_{k'\uparrow} + b^*_{k'\downarrow} a_{k\uparrow}) \\ + (\vec{S}_1 + \vec{S}_2)^- (a^*_{k\uparrow} b_{k'\downarrow} + b^*_{k'\uparrow} a_{k\downarrow}) \} + T_a \sum_{kk',\sigma} (1 + e^{i(\vec{k}-\vec{k}')\cdot\vec{R}}) (a^*_{k\sigma} b_{k'\sigma} + b^*_{k'\sigma} a_{k\sigma}) \,, \end{aligned} \tag{23}$$

$$\begin{aligned} H_{T2} = T_{J_a} \sum_{kk'} (1 - e^{i(\vec{k}-\vec{k}')\cdot\vec{R}}) \{ (\vec{S}_1 - \vec{S}_2)_z [(a^*_{k\uparrow} b_{k'\uparrow} - a^*_{k\downarrow} b_{k'\downarrow}) + (b^*_{k'\uparrow} a_{k\uparrow} - b^*_{k'\downarrow} a_{k\downarrow})] \\ + (\vec{S}_1 - \vec{S}_2)^+ (a^*_{k\downarrow} b_{k'\uparrow} + b^*_{k'\downarrow} a_{k\uparrow}) + (\vec{S}_1 - \vec{S}_2)^- (a^*_{k\uparrow} b_{k'\downarrow} + b^*_{k'\uparrow} a_{k\downarrow}) \} \,, \end{aligned} \tag{24}$$

$$H_{R1} = J_a \sum_{kk'} (1 + e^{i(\vec{k}-\vec{k}')\cdot\vec{R}}) [(\vec{S}_1 + \vec{S}_2)_z (a^*_{k\uparrow} a_{k'\uparrow} - a^*_{k\downarrow} a_{k'\downarrow}) + (\vec{S}_1 + \vec{S}_2)^+ (a^*_{k\downarrow} a_{k'\uparrow}) + (\vec{S}_1 + \vec{S}_2)^- (a^*_{k\uparrow} a_{k'\downarrow})], \tag{25}$$

$$H_{R2} = J_a \sum_{kk'} (1 - e^{i(\vec{k}-\vec{k}')\cdot\vec{R}}) [(\vec{S}_1 - \vec{S}_2)_z (a^*_{k\uparrow} a_{k'\uparrow} - a^*_{k\downarrow} a_{k'\downarrow}) + (\vec{S}_1 - \vec{S}_2)^+ (a^*_{k\downarrow} a_{k'\uparrow}) + (\vec{S}_1 - \vec{S}_2)^- (a^*_{k\uparrow} a_{k'\downarrow})] \,. \tag{26}$$

III. CALCULATION OF TUNNELING CURRENT (SECOND ORDER)

For calculating current across the barrier, we neglect multiple scattering by localized spins pairs since the impurity concentration is assumed to be low. The tunneling current j_{ab} can be written as

$$\begin{aligned} j_{ab} = e \sum_{I,M} P_{\vec{I},\vec{M}} \sum_{kk',\sigma\sigma'} \{ W_{k\sigma IM \to k'\sigma' I'M'} f(\epsilon^a_{k\sigma}) [1 - f(\epsilon^b_{k'\sigma'})] \} \\ - e \sum_{I'M'} P_{\vec{I}'\vec{M}'} \sum_{kk',\sigma\sigma'} \{ W_{k'\sigma' I'M' \to k\sigma IM} f(\epsilon^b_{k'\sigma'}) [1 - f(\epsilon^a_{k\sigma})] \} \,. \end{aligned} \tag{27}$$

$P_{\vec{I},\vec{M}}$ is the statistical probability that the impurity pair is having total spin equal to $\vec{I}$ and its z component M. $f(\epsilon_{k\sigma})$ is the Fermi-Dirac distribution function. Since no magnetic field has been applied, $f(\epsilon_k)$ will be the same for both spins and therefore we shall omit σ in $f(\epsilon^a_{k\sigma})$. $W_{k\sigma IM \to k'\sigma' I'M'}$ is the transition probability per unit time for a conduction electron to go from the state $|\vec{k}, \vec{\sigma}\rangle$ on the side a to the state $|\vec{k}', \vec{\sigma}'\rangle$ on side b with the localized spins undergoing $\vec{M} \to \vec{M}'; \vec{I} \to \vec{I}'$, such that

$$\vec{\sigma} + \vec{M} = \vec{\sigma}' + \vec{M}' \,. \tag{28}$$

$W_{k'\sigma' I'M' \to k\sigma IM}$ has similar meaning from side b to a.

We treat H' as a perturbation and following Kondo evaluate $W_{i,j}$ up to third order in H_{ij}. It is given by

$$W_{i,j} = \frac{2\pi}{\hbar} \left(\sum_{k \neq i} \frac{H'_{ik} H'_{kj} H'_{ji}}{E_i - E_j} + \text{c.c.} + |H_{ij}|^2 \right) \times \delta(E_i - E_j) \,, \tag{29}$$

where i, j, and k are initial, final, and intermediate states, respectively. E_i's are corresponding energies. The second term in the large parentheses represents the first Born approximation for the transition probability.

Various distinct physical processes for the transition of conduction electron with momentum $\vec{k}$ on side a to states with momentum $\vec{k}'$ on side b with their transition probabilities in the Born approximation are

$$\left. \begin{matrix} W^{(2)}_{k\uparrow 00 \to k'\uparrow 00} \\ \text{or} \\ W^{(2)}_{k\uparrow 10 \to k'\uparrow 10} \end{matrix} \right\} = (2\pi/\hbar) [\, |(1 + e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})|^2 \, T_a^{\,2}] \, \delta(\tilde{\epsilon}^a_k - \epsilon^b_{k'}), \tag{30}$$

$$\left.\begin{matrix} W^{(2)}_{k\uparrow 1\downarrow \to k'\uparrow 1\downarrow} \\ \text{or} \\ W^{(2)}_{k\uparrow 1\downarrow \to k'\uparrow 1\downarrow} \end{matrix}\right\} = (2\pi/\hbar)\left[\left|(1+e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})\right|^2 (T_a^2 + \tfrac{1}{4} T_{J_a}^2)\right] \delta(\tilde{\epsilon}^a_k - \epsilon^b_{k'}) , \tag{31}$$

$$\left.\begin{matrix} W^{(2)}_{k\uparrow 0\downarrow \to k'\uparrow 1\downarrow} \\ \text{or} \\ W^{(2)}_{k\uparrow 1\downarrow \to k'\uparrow 0\downarrow} \end{matrix}\right\} = (2\pi/\hbar)\left[\left|(1+e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})\right|^2 \tfrac{1}{2}(T_{J_a}^2)\right] \delta(\tilde{\epsilon}^a_k - \epsilon^b_{k'}) , \tag{32}$$

$$W^{(2)}_{k\uparrow 00\to k'\uparrow 10} = (2\pi/\hbar)\left[\left|(1-e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})\right|^2 \tfrac{1}{4}(T_{J_a}^2)\right] \delta(\tilde{\epsilon}^a_k + W - \epsilon^b_{k'}) , \tag{33}$$

$$W^{(2)}_{k\uparrow 10\to k'\uparrow 00} = (2\pi/\hbar)\left[\left|(1-e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})\right|^2 \tfrac{1}{4}(T_{J_a}^2)\right] \delta(\tilde{\epsilon}^a_k - W - \epsilon^b_{k'}) , \tag{34}$$

$$W^{(2)}_{k\uparrow 00\to k'\uparrow 11} = (2\pi/\hbar)\left[\left|(1-e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})\right|^2 \tfrac{1}{2}(T_{J_a}^2)\right] \delta(\tilde{\epsilon}^a_k + W - \epsilon^b_{k'}) , \tag{35}$$

$$W^{(2)}_{k\uparrow 1\downarrow\to k'\uparrow 00} = (2\pi/\hbar)\left[\left|(1-e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})\right|^2 \tfrac{1}{2}(T_{J_a}^2)\right] \delta(\tilde{\epsilon}^a_k - W - \epsilon^b_{k'}) . \tag{36}$$

Using (27) one obtains the expression for the current. We can split the current j_{ab} into two terms $j_1^{(2)}$ and $j_2^{(2)}$. $j_1^{(2)}$ represents current from elastic processes, while $j_2^{(2)}$ also includes current from inelastic processes. (The subscript ab on $j_{ab}^{(2)}$ has been dropped.) We find

$$j_1^{(2)} = \frac{2\pi e}{\hbar}\sum_{kk'}\left|(1+e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})\right|^2 (T_a^2 + \tfrac{3}{2} P_1 T_{J_a}^2) \times[f(\epsilon^a_k) - f(\epsilon^b_{k'})]\,\delta(\epsilon^a_k + eV - \epsilon^b_{k'}) , \tag{37}$$

$$\begin{aligned} j_2^{(2)} = \frac{2\pi e}{\hbar}\sum_{kk'}&\left[\left|(1-e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})\right|^2 \tfrac{3}{4}(T_{J_a}^2)\right] \\ &\times\Big(\{P_0(f(\epsilon^a_k))[1-f(\epsilon^b_{k'})] \\ &- P_1(f(\epsilon^b_{k'}))[1-f(\epsilon^a_k)]\}\,\delta(\epsilon^a_k + eV + W - \epsilon^b_{k'}) \\ &+\{P_1(f(\epsilon^a_k)[1-f(\epsilon^b_{k'})]) \\ &- P_0(f(\epsilon^b_{k'}))[1-f(\epsilon^a_k)]\}\,\delta(\epsilon^a_k + eV - W - \epsilon^b_{k'})\Big) . \end{aligned} \tag{38}$$

We note that for $eV < W$ and $T = 0$ the last term of (38) will be zero. This is to be expected on general physical grounds. An electron of energy ϵ on side a would have to tunnel into an energy state $\epsilon - W$ on side b. For $eV < W$ and $T = 0$, this process is forbidden by the exclusion principle. The above arguments are not restricted to second-order processes and are true to all orders in H'.

We are more interested in g, the conductance, which is given by

$$g = \left(\frac{\partial}{\partial V}\right) j . \tag{39}$$

If the sums over $\vec{k}$ and $\vec{k}'$ are replaced by integrals over energy, the derivative with respect to voltage is taken, and $-(\partial f/\partial\epsilon)(\epsilon - W)$ is replaced by $\delta(\epsilon - W)$ then one obtains

$$g_1^{(2)} = \frac{\partial}{\partial V} j_1^{(2)} = \frac{4\pi e^2}{\hbar}\,\rho^a(\epsilon_F)\,\rho^b(\epsilon_F)\left|(1+e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})\right|^2_{\text{av}} \times(T_a^2 + \tfrac{3}{2} P_1 T_{J_a}^2) , \tag{40}$$

$$\begin{aligned} g_2^{(2)} = \frac{\partial}{\partial V} j_2^{(2)} = \frac{4\pi e^2}{\hbar}\,\rho^a(\epsilon_F)\,\rho^b(\epsilon_F)&\left|(1-e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})\right|^2_{\text{av}} \\ \times\tfrac{3}{4}(T_{J_a}^2)\Big\{(P_0+P_1) + \tfrac{1}{2}(P_0-P_1)&\left[\tanh\left(\frac{eV+W}{2k_B T}\right)\right. \\ &\left.+\tanh\left(\frac{W-eV}{2k_B T}\right)\right]\Big\} . \end{aligned} \tag{41}$$

Now here $|(1+e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})|^2_{\text{av}}$ represents the average value of it for all $\vec{k}$ and $\vec{k}'$.

In the above analysis, we have made the assumption that the density of state $\rho(\epsilon)$ is constant and it has been removed from the integral sign and replaced by its value at $\epsilon = \epsilon_F$, i.e., by $\rho(\epsilon_F)$.

For the case $W = 0$ and $R \to \infty$, i.e., the two impurities are noninteracting and very far apart, the average value of $|(1+e^{i(\vec{k}-\vec{k}')\cdot\vec{R}})|^2 = 2$, and thus we get

$$g^{(2)}(4\pi e^2/\hbar)\,2\rho^a(\epsilon_F)\,\rho^b(\epsilon_F)\,(T_a^2 + \tfrac{3}{4} T_{J_a}^2) . \tag{42}$$

Dividing it by two, since we are having two impurities, we shall get the same expression for conductance as obtained by Appelbaum, with $\vec{S} = \frac{1}{2}$ for a single impurity.

IV. CALCULATION OF TUNNELING CURRENT (THIRD ORDER)

We proceed now to calculate the third-order term,

$$W^{(3)}_{i,j} = \frac{4\pi}{\hbar}\sum_{k\neq i}\frac{H'_{ik}H'_{kj}H'_{ji}}{E_i - E_k}\,\delta(E_i - E_j) , \tag{43}$$

where we have assumed that J_a, T_a, and T_{J_a} are real and combined the first and second terms of (29). For calculating $W^{(3)}_{ij}$, we shall be following

Kondo.[7] The transition from the state i to j by means of an intermediate state $\vec{k}$ may occur in two different ways. The electron in state i scatters into state k and then the electron in state k scatters into state j, or an electron in the intermediate state k first scatters into the final state j and then the electron in state i scatters into the hole left by the k electron. The terms which do not contain $f(\epsilon_q^a)$ will be dropped as they are very small. Further we shall replace $\sum_q \cdots$ by

$$P\int_{-E_0}^{E_0}\cdots\rho(\epsilon)\,d\epsilon\,. \tag{44}$$

That is, we restrict the principal-valued sum over intermediate states to an energy region of width $2E_0$ centered at the Fermi Energy ϵ_F (all energies are measured from ϵ_F).

Thus one obtains the following expressions for the transition probabilities considering only the terms which are proportional to $T_{J_a}{}^2 J_a$:

$$W^{(3)}_{k\uparrow 00\to q\downarrow 11\to k'\uparrow 10} = W^{(3)}_{k\uparrow 00\to q\uparrow 10\to k'\downarrow 11} = W^{(3)}_{k\uparrow 00\to q\downarrow 11\to k'\downarrow 11}$$

$$= -\tfrac{1}{8} T_{J_a}{}^2 J_a P_0 \sum_q \left(\frac{-f_q}{\epsilon_k - \epsilon_q + W}(2b + b_1) - \frac{f_q}{\epsilon_k - \epsilon_q}(2b - b_1) \right) \delta(\tilde{\epsilon}_k^a + W - \epsilon_{k'}^b)\,, \tag{45}$$

$$W^{(3)}_{k\uparrow 10\to q\downarrow 11\to k'\uparrow 00} = W^{(3)}_{k\uparrow 1\text{-}1\to q\uparrow 1\text{-}1\to k'\uparrow 00} = W^{(3)}_{k\uparrow 1\text{-}1\to q\downarrow 10\to k'\downarrow 00}$$

$$= -\tfrac{1}{8} T_{J_a}{}^2 J_a P_1 \sum_q \left(\frac{-f_q}{\epsilon_k - \epsilon_q - W}(2b + b_1) - \frac{f_q}{\epsilon_k - \epsilon_q}(2b - b_1) \right) \delta(\tilde{\epsilon}_k^a - W - \epsilon_{k'}^b)\,, \tag{46}$$

$$W^{(3)}_{k\uparrow 1\text{-}1\to q\downarrow 00\to k'\uparrow 1\text{-}1} = W^{(3)}_{k\uparrow 10\to q\uparrow 00\to k'\downarrow 11} = W^{(3)}_{k\uparrow 1\text{-}1\to q\downarrow 00\to k'\downarrow 10}$$

$$= -\tfrac{1}{8} T_{J_a}{}^2 J_a P_1 \sum_q \left[\left(-\frac{f_q}{\epsilon_k - \epsilon_q + W} - \frac{f_q}{\epsilon_k - \epsilon_q - W} \right)(2a - a_1) \right] \delta(\tilde{\epsilon}_k^a - \epsilon_{k'}^b)\,, \tag{47}$$

$$W^{(3)}_{k\uparrow 1\text{-}1\to q\downarrow 10\to k'\uparrow 1\text{-}1} = W^{(3)}_{k\uparrow 10\to q\downarrow 11\to k'\downarrow 11} = W^{(3)}_{k\uparrow 1\text{-}1\to q\uparrow 1\text{-}1\to k'\downarrow 10}$$

$$= -\tfrac{1}{8} T_{J_a}{}^2 J_a P_1 \sum_q \left[\left(-\frac{2f_q}{\epsilon_k - \epsilon_q} \right)(2a + a_1) \right] \delta(\tilde{\epsilon}_k^a - \epsilon_{k'}^b)\,, \tag{48}$$

where

$$a = (1 + e^{-i\vec{K}\cdot\vec{R}})(1 + e^{i\vec{k}\cdot\vec{R}}) = 2 + (e^{i\vec{k}\cdot\vec{R}} + e^{-i\vec{k}\cdot\vec{R}})\,, \tag{49}$$

$$b = (1 - e^{-i\vec{K}\cdot\vec{R}})(1 - e^{i\vec{k}\cdot\vec{R}}) = 2 - (e^{i\vec{k}\cdot\vec{R}} + e^{-i\vec{k}\cdot\vec{R}})\,, \tag{50}$$

$$\vec{K} = \vec{k} - \vec{k}'\,,\quad |\vec{k}| = |\vec{k}'| = |\vec{q}|\,,\quad |\vec{K}| = 2k\sin\tfrac{1}{2}\Theta\,. \tag{51}$$

Θ is the scattering angle between $\vec{k}$ and $\vec{k}'$ and the scattering can be considered as quasielastic. [The error is of order $(W + eV)/\epsilon_F$, and can be neglected since we are working with a weak interaction and small voltages.] Also, we have

$$a_1 = \alpha e^{-i\vec{q}\cdot\vec{R}} + \text{c.c.}\,,\quad b_1 = \beta e^{-i\vec{q}\cdot\vec{R}} + \text{c.c.}\,, \tag{52}$$

$$\alpha = 2(e^{i\vec{k}'\cdot\vec{R}} + e^{i\vec{k}\cdot\vec{R}})\,,\quad \beta = 2(e^{i\vec{k}'\cdot\vec{R}} - e^{i\vec{k}\cdot\vec{R}})\,. \tag{53}$$

The average values of the above quantities for all possible orientations of $\vec{R}$ are given by

$$\bar{a} = (1/4\pi)\int d\Omega_{\vec{R}}\, a = 2 + [8/(2kR)^4]\{2(2kR)\sin(2kR) - [(2kR)^2 - 2]\cos(2kR) - 2\}\,, \tag{54}$$

$$\bar{b} = (1/4\pi)\int d\Omega_{\vec{R}}\, b = 2 - [8/(2kR)^4]\{2(2kR)\sin(2kR) - [(2kR)^2 - 2]\cos(2kR) - 2\}\,, \tag{55}$$

$$\bar{a}_1 = 4(\bar{a} - 2)\,, \tag{56}$$

$$\bar{b}_1 = 0\,. \tag{57}$$

The above terms represent the interference between reflected and transmitted exchange-scattered current. Because we are mainly interested in anomalous temperature- and voltage-dependent terms, we determine if there are any terms proportional to $T_a^2 J_a$ or $T_a T_{J_a} J_a$ which exhibit anomalous temperature and voltage dependence. Terms proportional to $T_a^2 J_a$ clearly cannot be of importance as only one exchange scattering is involved. The terms proportional to $T_{J_a} T_a J_a$ will give the following contributions:

(a) $W^{(3)}_{k\uparrow 00\to k'\uparrow 00}$ through the intermediate step $\vec{q}\downarrow 11$ and $\vec{q}\uparrow 10$ have the total contribution

$$-\tfrac{3}{8} T_a T_{J_a} J_a P_0 \sum_q \left[\left(\frac{-f_q}{\epsilon_k - \epsilon_q + W} + \frac{f_q}{\epsilon_k - \epsilon_q - W} \right)(2a - a_1) \right] \times \delta(\tilde{\epsilon}_k^a - \epsilon_{k'}^b)\,. \tag{58}$$

(b) $W^{(3)}_{k\uparrow 10\to k'\uparrow 10}$ through the intermediate step

$\vec{q}\downarrow 00$ and $W^{(3)}_{k\uparrow 1-1\to k'\uparrow 1-1}$ through the intermediate step $\vec{q}\downarrow 00$, etc., will have the total contribution

$$\tfrac{3}{8} T_a T_{J_a} J_a P_1 \sum_q \left[\left(\frac{-f_q}{\epsilon_k - \epsilon_q + W} + \frac{f_q}{\epsilon_k - \epsilon_q - W}\right)(2a - a_1)\right] \times \delta(\epsilon_k^a - \epsilon_{k'}^b) \ . \quad (59)$$

In the Appelbaum[5] calculation, the terms proportional to $T_{J_a} T_a J_a$ are shown to be identically zero when no magnetic field is applied. With

$$g^a(\omega) \equiv \sum_q \frac{f(\epsilon_q^a)}{\epsilon_q^a - W} = P\int_{-E_0}^{E_0} \frac{f(\epsilon)\,\rho^a(\epsilon)\,d\epsilon}{\epsilon - W} \ , \quad (60)$$

the current in third order can be given as (b_1 is taken to be zero)

$$j_1^{(3)} = \left(-\frac{2\pi e}{\hbar}\right)\left(\sum_{kk'} \tfrac{3}{4} T_{J_a}{}^2 J_a P_1 (2a + a_1)\,[2g^a(\epsilon_k)]\,[f(\epsilon_k) - f(\epsilon_{k'})]\,\delta(\tilde{\epsilon}_k^a - \epsilon_{k'}^b)\right) , \quad (61)$$

$$\begin{aligned} j_2^{(3)} = \left(-\frac{2\pi e}{\hbar}\right)\Bigg[& (\tfrac{3}{2} T_{J_a}{}^2 J_a b)\Bigg(\sum_{kk'} \{[P_0 g^a(\epsilon_k + W) + P_0 g^a(\epsilon_k)][f(\epsilon_k^a) - f(\epsilon_k^a) f(\epsilon_{k'}^b)] - [P_1 g^a(\epsilon_k) + P_1 g^a(\epsilon_k + W)] \\ & \times [f(\epsilon_{k'}^b) - f(\epsilon_k^a) f(\epsilon_{k'}^b)]\}\,\delta(\tilde{\epsilon}_k^a + W - \epsilon_{k'}^b) + \sum_{kk'} \{[P_1 g^a(\epsilon_k - W) + P_1 g^a(\epsilon_k)][f(\epsilon_k^a) - f(\epsilon_k^a) f(\epsilon_{k'}^b)] \\ & - [P_0 g^a(\epsilon_k) + P_0 g^a(\epsilon_k - W)][f(\epsilon_{k'}^b) - f(\epsilon_k^a) f(\epsilon_{k'}^b)]\}\,\delta(\epsilon_k^a - W - \epsilon_{k'}^b)\Bigg) \\ & + \sum_{kk'} \tfrac{3}{4} T_{J_a}{}^2 J_a P_1 \{(2a - a_1)[g^a(\epsilon_k + W) + g^a(\epsilon_k - W)]\}[f(\epsilon_k) - f(\epsilon_{k'})]\,\delta(\tilde{\epsilon}_k^a - \epsilon_{k'}^b) \\ & + \sum_{kk'} \tfrac{3}{4} T_{J_a} J_a T_a (P_0 - P_1)\{(2a - a_1)[g^a(\epsilon_k + W) - g^a(\epsilon_k - W)]\}[f(\epsilon_k) - f(\epsilon_{k'})]\,\delta(\tilde{\epsilon}_k^a - \epsilon_{k'}^b)\Bigg] \ . \end{aligned} \quad (62)$$

Replacing $\sum_{kk'} \cdots$ by $\int\int d\epsilon\, d\epsilon' \rho^a(\epsilon)\rho^b(\epsilon') \cdots$ and performing the integral over ϵ', we take the derivative with respect to V. With

$$F(\omega) = -\int_{-\infty}^{\infty} g(\epsilon)\,\frac{\partial f}{\partial \epsilon}(\epsilon - W)\,d\epsilon \ , \quad (63)$$

we obtain for $g^{(3)}$

$$g_1^{(3)} = (-4\pi e^2/\hbar)\,\tfrac{3}{4} T_{J_a}{}^2 J_a P_1 \rho^a(\epsilon_F)\,\rho^b(\epsilon_F)\,(2a + a_1)\,2F(eV) \ , \quad (64)$$

$$\begin{aligned} g_2^{(3)} = \left(-\frac{4\pi e^2}{\hbar}\right)\Bigg(& \tfrac{3}{2} T_{J_a}{}^2 J_a b\,\rho^a(\epsilon_F)\,\rho^b(\epsilon_F)\Bigg\{F(eV)\left[(P_0 + P_1) + \left(\frac{P_0 - P_1}{2}\right)\left(\tanh\frac{eV + W}{2k_B T} + \tanh\frac{W - eV}{2k_B T}\right)\right] \\ & + F(eV + W)\left[P_1 + \left(\frac{P_0 - P_1}{2}\right)\left(\tanh\frac{eV + W}{2k_B T} + 1\right)\right] + F(eV - W)\left[P_0 + \left(\frac{P_0 - P_1}{2}\right)\left(\tanh\frac{W - eV}{2k_B T} - 1\right)\right]\Bigg\} \\ & + \tfrac{3}{4} T_{J_a}{}^2 J_a P_1 \rho^a(\epsilon_F)\,\rho^b(\epsilon_F)\,[(2a - a_1)\,F(eV - W) + (2a - a_1)\,F(eV + W)] \\ & + \tfrac{3}{4} T_{J_a} T_a J_a (P_0 - P_1)\,\rho^a(\epsilon_F)\,\rho^b(\epsilon_F)\,[(2a - a_1)\,F(eV - W) - (2a - a_1)\,F(eV + W)]\Bigg) . \end{aligned} \quad (65)$$

V. DISCUSSION

First we consider the various limiting cases of the expressions derived for conductance above. It can be easily seen that the expressions (40), (41), (64), and (65) reduce to those obtained by Appelbaum,[5] for (a) $W/k_B T \to 0$, $R \to \infty$; (b) $W/k_B T \to \infty$, $R \to 0$; (c) $W/k_B T \to -\infty$, $R \to 0$ and in the absence of any external magnetic field.

The interesting point is now to examine the voltage and temperature variation of conductivity in the general case. For doing numerical calculations, we assume the concentration of the impurities to be low, so that the interaction between impurities is of the order of 10^{-4} eV and $\bar{a}$, $\bar{b}$, $\bar{a}_1$, $\bar{b}_1$ can be replaced approximately by their values when $R \to \infty$. T_a is taken to be twice that of T_J, and E_0 equals to 10 meV for the present calculations.

In Figs. 2 and 3, we have plotted the variation of conductivity in second order with respect to biasing voltage and temperature, respectively.

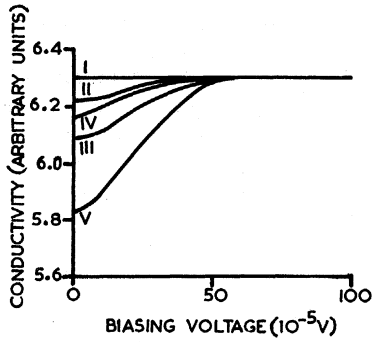

FIG. 2. Variation of conductivity with applied biasing voltage (second order). Temperature is 1.16 °K. Curves I, II, III, IV, and V correspond to values of $W=0$, 10^{-4}, 2×10^{-4}, -10^{-4}, and -2×10^{-4} eV, respectively.

We observed a dip at zero bias. The amount of dip increases with increase in interaction and it is more for antiferromagnetic coupling in comparison to ferromagnetic coupling between the impurity spins. This is in contrast to Appelbaum's result where the $g^{(2)}$ is found to be constant with respect to biasing voltage and temperature in the absence of any magnetic field.

In Figs. 4 and 5, we have plotted the conductivity in third order with respect to biasing voltage and temperature, respectively. In the case of antiferromagnetic coupling between impurities, we observe maxima at slightly different biasing voltages and temperatures from that found in the case of noninteracting impurities.

The ideal behavior of the metal-insulator-metal junction at low temperature for fairly low voltage (up to 500 meV) has been discussed by many authors, [11–13] without the resonance term. In Fig. 6, we combine our results with interacting paramagnetic impurities for total conductance, with ideal

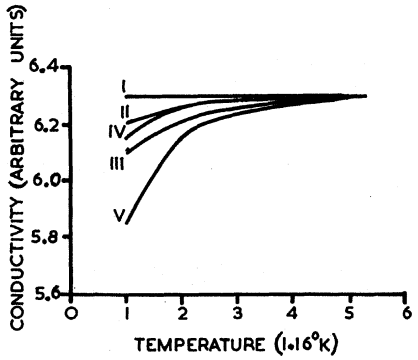

FIG. 3. Variation of conductivity with temperature (second order). Biasing voltage is zero. Curves I, II, III, IV, and V correspond to values of $W=0$, 10^{-4}, 2×10^{-4}, -10^{-4}, and -2×10^{-4} eV, respectively.

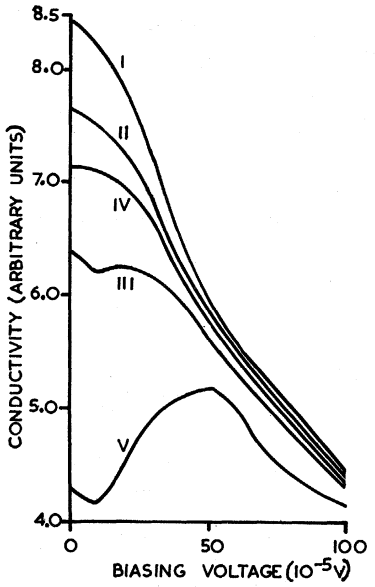

FIG. 4. Variation of conductivity with applied biasing voltage (third order). Temperature is 1.16 °K. Curves I, II, III, IV, and V correspond to values of $W=0$, 10^{-4}, 2×10^{-4}, -10^{-4}, and -2×10^{-4} eV, respectively.

behavior in case of T_a-I-M for various strengths of interaction between the impurities.

One finds that as the interaction between the impurity spins is switched on, the resistance shows a maximum for zero bias instead of a dip as in the case of noninteracting impurities. This result is in agreement qualitatively with the experimental results reported by Mezei, [14] for the case of

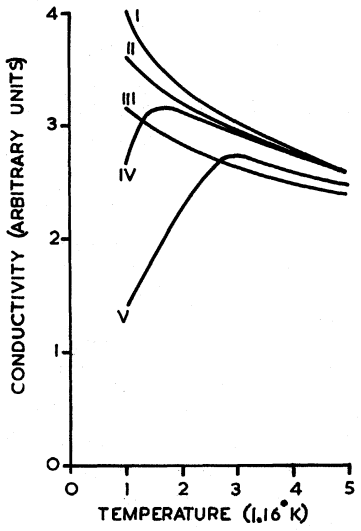

FIG. 5. Variation of conductivity with temperature (third order). Biasing voltage is zero. Curves I, II, III, IV, and V correspond to values of $W=0$, 10^{-4}, 2×10^{-4}, -10^{-4}, and -2×10^{-4} eV, respectively.

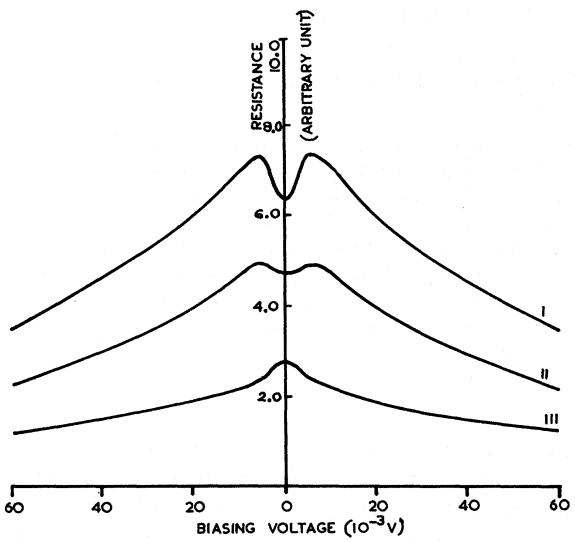

FIG. 6. Variation of conductivity with applied biasing voltage. Temperature is 1.16 °K. Curves I, II, and III correspond to values of $W=0$, -10^{-4}, and -2×10^{-4} eV, respectively.

Al-*I*-Al junction. It is found by Mezei that with the gradual increase of Cr impurities in the barrier region, one obtains a resistance maximum at zero bias instead of conductance maximum as in the case of very low concentration of impurities. It seems that with the increase in concentration of impurities, the impurity spins start getting ordered either ferromagnetically or antiferromagnetically as considered in the present paper. A rough numerical estimate for the concentration at which this magnetic ordering may take place, gives a value of 1 at.% for impurity concentration, when only the RKKY-type interaction is assumed to be effective. Thus, when the concentration of impurities is larger than this critical value, the interaction mechanism envisaged here will be effective and the conductance will be proportional to the concentration of impurity pairs. It is to be noted that the earlier attempts to explain the resistance maximum at zero bias in other systems[5] will not be applicable here as they imply ferromagnetic coupling between *s* and *d* electrons, whereas Mezei finds both the dip in conductance and peak in conductance for different concentration of Cr impurities in the same tunnel junction. However, the magnitude of the resistance maximum as observed by Mezei cannot be explained very well by our results.

It is expected that in the presence of an external magnetic field, the conclusions deduced above about the conductance will remain valid if the impurity pair is antiferromagnetically ordered unless the strength of the field is such as to break the pair effectively. However, when the pair is ferromagnetically ordered, the zero bias conductance, like that in Appelbaums's calculations, will show two additional peaks.

In the present calculations, the dependence of the conductance on the position of the magnetic impurities is not considered. This has been investigated recently by Appelbaum and Brinkman,[15] for a single magnetic impurity in the interface. Furthermore, a perturbation theoretical approach is followed and it is expected that this approach provides a good account of the problem considered here. However, it is desirable to treat the problem by more advanced techniques,[15–17] particularly in the case of strong coupling between conduction electrons and localized impurities. Such improvements, as well as the detailed investigation of the dependence of conductance on concentration by considering the average of the contributions from all possible pairs, is in progress.

ACKNOWLEDGMENT

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[1]C. B. Duke, *Tunneling in Solids* (Academic, New York, 1969), Suppl. 10.

[2]A. F. G. Wyatt, Phys. Rev. Letters 13, 401 (1964).

[3]D. J. Kim, Phys. Letters 18, 215 (1966).

[4]P. W. Anderson, Phys. Rev. Letters 17, 95 (1966).

[5]J. Appelbaum, Phys. Rev. 154, 633 (1967); Phys. Rev. Letters 17, 91 (1966).

[6]J. Solyom and A. Zawadowski, Physik. Kondensierten Materie 7, 325 (1968).

[7]J. Kondo, Progr. Theoret. Phys. (Kyoto) 32, 37 (1964).

[8]M. T. Beal Monod, Phys. Rev. 178, 874 (1969).

[9]M. A. Rudermann and C. Kittel, Phys. Rev. 96, 99 (1954); T. Kasuya, Progr. Theoret. Phys. (Kyoto) 16, 45 (1956); K. Yosida, Phys. Rev. 106, 893 (1957).

[10]M. H. Cohen, L. M. Falicov, and J. C. Phillips, Phys. Rev. Letters 8, 316 (1962).

[11]J. G. Simmons, J. Appl. Phys. 34, 238 (1963).

[12]H. P. Kanauss and R. A. Breslow, Proc. IRE 50, 1834 (1962).

[13]*Tunneling Phenomena in Solids*, edited by E. Burstein and S. Lundgvist (Plenum, New York, 1969).

[14]F. Mezei, Phys. Letters 25A, 354 (1967).

[15]J. A. Appelbaum and W. F. Brinkman, Phys. Rev. B 2, 907 (1970).

[16]Y. Nagaoka, Phys. Rev. 138, A1112 (1965).

[17]J. A. Appelbaum, J. C. Phillips, and G. Tzouras, Phys. Rev. 160, 554 (1967).